Name: **Version #1**

Instructor: Annette McP

Math 10120 Exam 2
Oct. 12, 2021.

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off all cellphones and electronic devices.
- Calculators **are** allowed.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all 11 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)
2.	(•)	(b)	(c)	(d)	(e)
3.	(•)	(b)	(c)	(d)	(e)
4.	(•)	(b)	(c)	(d)	(e)
5.	(•)	(b)	(c)	(d)	(e)
6.	(•)	(b)	(c)	(d)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)
9.	(•)	(b)	(c)	(d)	(e)
10.	(•)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	
11.	
12.	
13.	
14.	
15.	
Total	

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2	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
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10.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	________
11.	________
12.	________
13.	________
14.	________
15.	________
Total	________

Multiple Choice

1.(5pts) Two four-sided dice are rolled, and the product of the numbers on their uppermost faces is recorded. What is the sample space for this experiment?

(a) $\{1, 2, 3, 4, 6, 8, 9, 12, 16\}$

(b) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$

(c) $\{2, 3, 4, 5, 6, 7, 8\}$

(d) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

(e) $\{4, 8, 12, 16\}$

1, 2, 3, 4 times 1 → {1, 2, 3, 4}
1, 2, 3, 4 times 2 → {2, 4, 6, 8}
1, 2, 3, 4 times 3 → {3, 6, 9, 12}
1, 2, 3, 4 times 4 → {4, 8, 12, 16}
union = (a)

2.(5pts) A sample space consists of 7 simple outcomes $\{a, b, c, d, e, f, g\}$. The probabilities are

$\Pr(a)$	$\Pr(b)$	$\Pr(c)$	$\Pr(d)$	$\Pr(e)$	$\Pr(f)$	$\Pr(g)$
0.10	0.16	0.15	0.19	0.08	0.12	0.20

What is $\Pr(\{c, d, g\})$? $= .15 + .19 + .20 = .54$

(a) 0.54 (b) 0.35 (c) 0.00144 (d) 0.65 (e) 0.429

3.(5pts) A coin is flipped 25 times and the resulting sequence of heads and tails is observed. What is the probability that the resulting sequence will have exactly 5 tails?

(a) $\dfrac{C(25,5)}{2^{25}}$ (b) $\dfrac{5}{2^{25}}$ (c) $\dfrac{2^5}{2^{25}}$ (d) $\dfrac{5}{25}$ (e) $\dfrac{C(25,5)}{C(25,25)}$

$$Pr(\text{Ex } 5\ T\text{'s}) = \frac{\#\text{ seq. with Ex } 5T\text{'s}}{\text{TOTAL } \#\text{ Seqs}} = \frac{C(25,5)}{2^{25}}$$

4.(5pts) A box contains 20 numbered marbles. Five of the marbles are red (numbered 1-5), five are blue(numbered 6-10) and 10 are green (numbered 11-20). A random sample of 10 marbles is taken from the box. What is the probability that the sample will contain at least one red marble?

(a) $1 - \dfrac{C(15,10)}{C(20,10)}$ (b) $\dfrac{5}{C(20,10)}$ (c) $1 - \dfrac{C(15,9)}{C(20,10)}$

(d) $1 - \dfrac{5}{C(20,10)}$ (e) $\dfrac{C(5,1)C(15,9)}{C(20,10)}$

TOT = 20

5R	5B	10G

Sample Size = 10

Pr(at Least 1R)

$= 1 - P(0R)$

$= 1 - \dfrac{C(15,10)}{C(20,10)}$ ← # samples 0R ; ↖ Total # samples

5.(5pts) Abigail goes to the grocery store each Saturday to stock up on Aero chocolate bars and PG Tips tea. On any given Saturday, the probability that the store has Aero chocolate bars in stock is 0.5, the probability that they have PG Tips tea in stock is 0.7 and the probability that they have both in stock is 0.4. What is the probability that the grocery store will have neither Aero bars nor PG Tips tea in stock when Abigail goes to shop next Saturday?

A P

(a) 0.2 (b) 0.8 (c) 0.1 (d) 0.9 (e) 0.5

$P(A) = 0.5$ $P(P) = 0.7$

$P(A \cap P) = 0.4$

Q What is $P((A \cup P)')$

$P(A \cup P) = P(A) + P(P) - P(A \cap P)$

$= 0.5 + 0.7 - 0.4 = 0.8$

$P((A \cup P)') = 1 - 0.8 = 0.2$

6.(5pts) In an experiment, you flip a coin 20 times and write down the sequence of heads and tails. What is the probability of the event "at least two heads appear in the sequence"?

(a) $1 - \frac{21}{2^{20}}$ (b) $\frac{21}{2^{20}}$ (c) $\frac{C(20,2)}{2^{20}}$

(d) $1 - \frac{39}{2^{20}}$ (e) $\frac{2^{20}}{C(20,2)}$

$P(\text{at Least } 2H) = 1 - P(0H \text{ or } 1H)$

$= 1 - \frac{C(20,0) + C(20,1)}{2^{20}}$

$= 1 - \frac{1+20}{2^{20}} = 1 - \frac{21}{2^{20}}$

$P(E) = 1 - 3/10 = \frac{7}{10}$

7.(5pts) Let E and F be two events in the same sample space S. If $Pr(E') = \frac{3}{10}$, $Pr(F) = \frac{6}{10}$, and $Pr(E \cap F) = \frac{1}{10}$, then what is $Pr(F|E)$?

(a) $\frac{1}{7}$ (b) $\frac{7}{100}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) $\frac{9}{50}$

$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{1/10}{7/10} = \frac{1}{7}.$

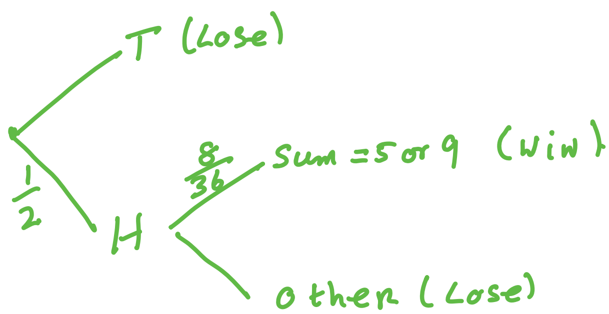
8.(5pts) In a carnival game, you first toss a coin. If you get a tail, you lose and the game is over. If you get a head, you roll a pair of six sided dice. If the sum of the numbers on the pair of dice is 5 or 9 you win, otherwise you lose. What is the probability that you win if you play this game? (A tree diagram might help)

(a) 4/36 (b) 1/12 (c) 1/2 (d) 8/36 (e) 26/36

Roll 2 dice. Outcomes with sum = 5 or 9

$\downarrow$

[(1,4) (2,3) (3,2) (4,1) (3,6) (4,5) (5,4) (6,3)]



$P(win) = \frac{1}{2} \cdot \frac{8}{36} = \frac{4}{36} = \boxed{\frac{1}{9}}$

9.(5pts) An electronic device contains 5 transistors working independently. The probability that a transistor will fail within 5000 hrs. is 0.03. What is the probability that at least one of the transistors in the device will fail within 5000 hrs.?

(a) $1-(0.97)^5$ (b) $(0.97)^5$ (c) $(0.03)^5$

(d) $1-(0.03)^5$ (e) 0

P(at Least 1 F) = 1 − P(No F)

$= 1-(0.97)^5$

10.(5pts) Suppose $\Pr(E)=0.4$, $\Pr(F)=0.2$ and $\Pr(E\cup F)=0.6$. Which of the statements below is correct?

(a) E and F are mutually exclusive but not independent.

(b) E and F are neither independent nor mutually exclusive.

(c) E and F are independent but not mutually exclusive.

(d) E and F are independent and mutually exclusive.

(e) Neither Independence nor mutual exclusivity can be determined from the given data.

M.E. if $P(E\cup F) = P(E)+P(F)$

ie $0.6 = 0.4+0.2$ ✓ TRue

Indep if $P(E\cap F) = P(E)P(F) = (0.4)(0.2)$

$= 0.08$

$P(E\cap F) = P(E)+P(F)-P(E\cup F)$

$= 0.4+0.2-0.6 = 0$ NOT Indep

For Questions 11-14, you may express your answers using the notation for permutations, combinations, powers and factorials, where appropriate

11.(12pts) Recall that a poker hand is a hand of 5 cards dealt randomly from a deck of 52. The deck has 4 suits (Hearts, Diamonds, Clubs and Spades) and 13 denominations (A's, K's, Q's, ..., 2's).

(a) What is the probability of getting a poker hand with 4 kings?

$$P(4K\text{'s (+ another card)}) = \frac{\#\text{ Hands with }4K\text{'s}}{\text{Total }\#\text{ Hands}}$$

$$= \frac{C(4,4)\cdot 48}{C(52,5)} = \frac{48}{C(52,5)}$$

(b) What is the conditional probability of getting a poker hand that consists entirely of hearts given that at least one of them is heart?

all H's $\cap$ at least 1H
= all H's
(since "all H's" is a subset of "at least 1H").

$$P(\text{all H's} \mid \text{at least 1H})$$

$$= \frac{P(\text{all H} \cap \text{at least 1H})}{P(\text{at least 1H})} = \frac{P(\text{all H})}{P(\text{at least 1H})}$$

$P(\text{at least 1H}) = 1 - P(0H)$

$$= \boxed{\frac{C(13,5)/C(52,5)}{1 - C(39,5)/C(52,5)}}$$

(c) What is the probability of getting 2 cards from one denomination and three from another?

$$= \frac{\#\text{ ways to choose 2 cards from one denom and 3 from another}}{\#\text{ hands}}$$

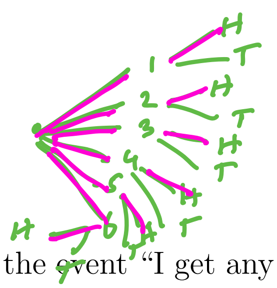
$$= \frac{13\cdot 12\, C(4,3)\, C(4,2)}{C(52,5)}.$$

12.(12pts) I have one six-sided die (with sides labled 1-6) and one coin. Suppose I roll the die and flip the coin, and I record the number I get from the die and whether I get heads or tails when I flip the coin.

(a) What is the sample space for this experiment? Write out the elements as a list of equally likely outcomes <u>or</u> represent the outcomes as paths on a tree diagram.

$\{(1,H), (2,H), (3,H), (4,H), (5,H), (6,H), (1,T), (2,T), (3,T), (4,T), (5,T), (6,T)\}$.

or



(b) Let E be the event "I get any number on the die and I get heads on the coin". Which subset of the sample space corresponds to the event E? Write out the elements of this subset of your equally likely sample space <u>or</u> highlight the corresponding paths on your tree diagram.

$E = \{(1,H), (2,H), (3,H), (4,H), (5,H), (6,H)\}$

(c) What is $Pr(E)$? $= \frac{\# E}{\# S.S.} = \frac{6}{12} = \frac{1}{2}$.

or From T. diagram

$P(E) = 6\left(\frac{1}{6} \cdot \frac{1}{2}\right) = \frac{1}{2}$

(d) Let F be the event "I get an even number on the die and tails on the coin". Are E and F mutually exclusive? Make sure to justify your answer.

yes since $E \cap F = \phi$.

$F = \{(2,T), (4,T), (6,T)\}$.

Initials: ____________

13.(12pts) A group of 100 students were asked about their preferred dining hall and their dorm. The results of this study are recorded in the following table:

	Morrisey	Farley	Walsh	
North Dining Hall	15	12	14	41
South Dining Hall	25	31	3	59
	40	43	17	100

Let F be the event "lives in Farley," and let N be the event "prefers North Dining Hall."

(a) What is the probability that a randomly selected student from the group lives in Farley? That is, what is $Pr(F)$?

$P(F) = \frac{43}{100}$

(b) Given that the person selected lives in Farley, what is the probability that the person also prefers North Dining Hall? That is, what is $Pr(N|F)$?

$P(N|F) = \frac{\# N\cap F}{\# F} = \frac{12}{43}$

(c) Are the events F and N independent? Make sure to justify your answer.

$P(N) = \frac{41}{100} \neq \frac{12}{43} = P(N|F)$

Therefore F and N are not indep

(d) Are the events F and N mutually exclusive? Make sure to justify your answer.

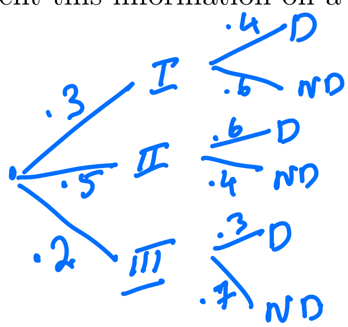
No $\# N\cap F = 12 \neq 0$

so N and F are not M.E.

14.(12pts) Glass ornaments are produced by three machines in a factory, creatively called Machine I, Machine II and Machine III. The table below shows the proportion of the output for which each is responsible and the probability that an ornament chosen at random from their respective outputs is defective.

Machine	Proportion of Output	Pr(defective)
Machine I	0.3	0.4
Machine II	0.5	0.6
Machine III	0.2	0.3

(a) Represent this information on a tree diagram.



An experiment consists of choosing a glass ornament at random from the factory's output.
Let $M1$ be the event that the ornament was produced by Machine I
Let $M2$ be the event that the ornament was produced by Machine II
Let $M3$ be the event that the ornament was produced by Machine III
Let D be the event that the ornament is defective

(b) What is $Pr(D)$, the probability of D?

$P(D) = (.3)(.4) + (.5)(.6) + (.2)(.3) = .12 + .3 + .06$
$= .48$

(c) What is $Pr(D \cap M1)$?

$(.3)(.4) = .12$

(d) What is $Pr(D|M1)$? $= P(D \cap M1) / P(M1) = \frac{(.3)(.4)}{.3} = \frac{.12}{.3} = .4$

(e) What is $Pr(M1|D)$?

$= \frac{P(D \cap M1)}{P(D)} = \frac{.12}{.48} = \frac{1}{4}$

15.(2pts) You will be awarded these two points if your name appears in CAPITALS on the front of your exam and you mark your answers on the front page with an X through your answer choice like so: (a) (<u>not</u> an O around your answer choice) . You may also use this page for

ROUGH WORK